Fundamental Theorems of Calculus/Trig Formulas/Derivatives

In a Nut Shell: The Fundamental Theorems of Calculus show that differentiation and integration are inverse processes. It establishes the connection between differential and integral calculus. The Fundamental Theorems of Calculus have two parts.

The Fundamental Theorem of Calculus (Part 1)

If f is continuous on [a, b], then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \le x \le b$$

is continuous on [a, b] and differentiable on (a, b), and $g'(x) = dg(x)/dx = f(x)$

The Fundamental Theorem of Calculus (Part 2)

If f is continuous on [a, b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f(x), that is, a function such that $F'(x) = dF/dx = f$.

Finding Derivatives of Integrals

$$F(x) = \int_{u(x)}^{v(x)} f(t)\,dt$$

where v(x) is the upper limit of integration

where u(x) is the lower limit of integration

Strategy: Take derivative **using the chain rule:**

$$dF(x)/dx = f(v(x))\,dv(x))/dx - f(u(x))\,du(x))/dx$$ **Equation A.**

Example 1 First do the integration, then take derivative of the result.

$$F(x) = \int_{t=0}^{t=x^2} \sin(t)\,dt = -\cos(t)\Big|_0^{x^2} = -[\cos(x^2) - 1]$$

$dF(x)/dx = \sin(x^2)(2x) = 2x \sin(x^2)$ (Result)

Next apply equation A.

$dF(x)/dx = \sin(x^2)(2x) - \sin(0)(0) = 2x \sin(x^2)$

(Note you get the same result.)

Example 2 **Using the chain rule given above in equation A**

$G(x) = \int_{e^{-x}}^{e^{x}} \ln(t)\, dt$ $f(t) = \ln(t)$, $v(x) = e^{x}$, $u(x) = e^{-x}$, $dv/dx = e^{x}$, $du/dx = -e^{-x}$

Apply $dG(x)/dx = f(v(x))\, dv(x))/dx - f(u(x))\, du(x))/dx$ $f(v) = x$, $f(u) = -x$

$dG(x)/dx = \ln(e^{x})\, e^{x} - \ln(e^{-x})(-e^{-x})$

$dG(x)/dx = x e^{x} - [-x(-e^{-x})] = x e^{x} - xe^{-x}$ (Result)

In a Nut Shell: Integrals may involve trig functions or combinations of trigonometric functions. Also, integrals may involve rational functions which when transformed yield trigonometric functions. In these cases, you must know the following trigonometric formulas as well as their derivatives.

Trig Formulas	Derivatives
$\sin^2 x + \cos^2 x = 1$	$d/dx(\sin x) = \cos x$
$\tan^2 x + 1 = \sec^2 x$	$d/dx(\cos x) = -\sin x$
$\cot^2 x + 1 = \csc^2 x$	$d/dx(\tan x) = \sec^2 x$
$\sin 2x = 2 \sin x \cos x$	$d/dx(\cot x) = -\csc^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$d/dx(\sec x) = \sec x \tan x$
$\cos^2 x = (1 + \cos 2x)/2$	
$\sin^2 x = (1 - \cos 2x)/2$	

More Trig Formulas

$\sin x \cos y = (1/2)[\sin(x+y) + \sin(x-y)]$
$\cos x \sin y = (1/2)[\sin(x+y) - \sin(x-y)]$
$\cos x \cos y = (1/2)[\cos(x+y) + \cos(x-y)]$
$\sin x \sin y = (1/2)[\cos(x-y) - \cos(x+y)]$

Pascals' Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

etc.

Note: Pascal's Triangle gives the binomial coefficients.

i.e. $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$