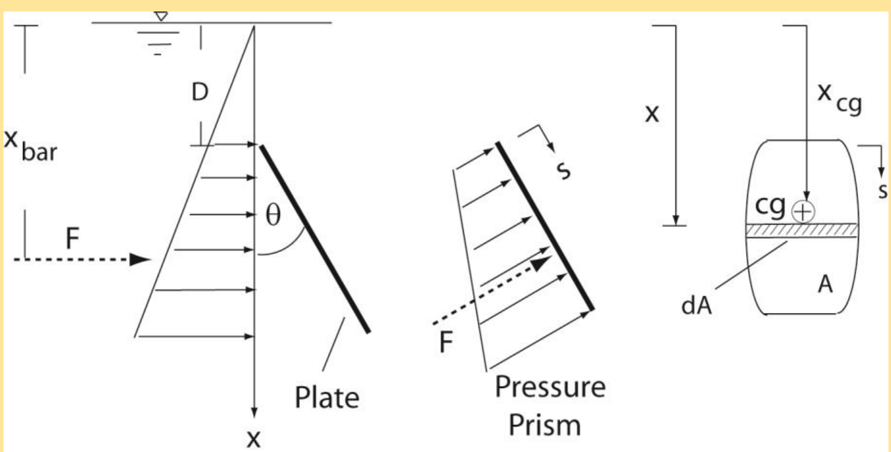
Hydrostatic Forces on Submerged Plates

Key Concepts: Submerged plates are subjected to hydrostatic pressure forces. Pressure increases linearly with depth. By integration it can be shown that **the magnitude of the hydrostatic pressure force** equals the pressure at the centroid of the plate times the area of the plate.

The location of the total hydrostatic pressure force is at the center of pressure which is at a point below the centroid of the plate. It can be calculated as well.

In a Nutshell: Plates submerged in liquids experience pressure forces. Pressure varies linearly with depth and is isotropic (same in all directions at a point in the liquid). The method of integration will be used to find the resultant hydrostatic pressure force.

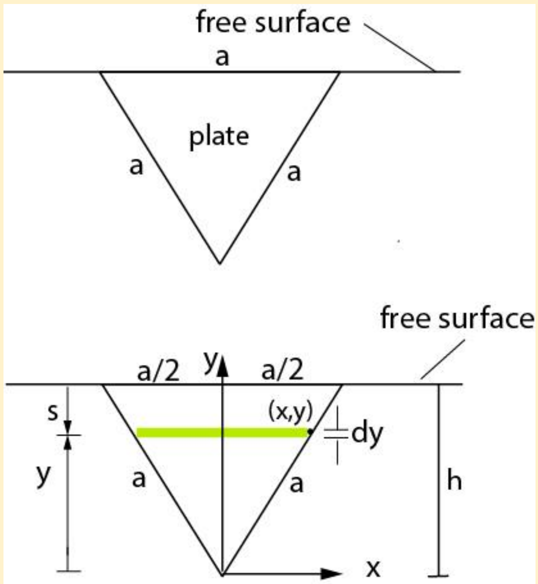
Consider a plate of arbitrary shaped plate submerged at an angle θ at a depth D below the free surface of a liquid as shown in the figure below. Denote the specific weight of the liquid by γ. Common units for specific weight include lb/ft^3 or n/m^3. Specific weight is the product of mass density, say ρ $slugs/ft^3$ or kg/m^3 times gravity, g ft/sec^2 or m/sec^2 .



The elemental pressure force dF acting on the element of area dA is dF = PdA.
Now the pressure, P, varies linearly with depth, x. So P = γ x and the differential pressure force is dF = = γ x dA . So integration gives the total hydrostatic pressure force, F.

$$F = \int \gamma\, x\, dA$$

Example: Find the total hydrostatic force on the equilateral plate with each side of length, a. The plate is submerged vertically in a liquid with mass density, ρ, and gravity, g as shown in the top figure below.



Strategy:

1. Set up a convenient coordinate system for the geometry of the plate, picking convenient coordinates x and y and a coordinate for depth, say s.
2. Identify the element of area, dA, upon which the pressure, P, acts.
3. Form the element of force, say dF = PdA.
4. Identify the limits of integration and integrate to obtain the total hydrostatic force.

Calculation:

The pressure varies linearly so $P = \rho g s$ where s is the depth to an arbitrary element

The element of force, $dF = P\,dA$ where $dA = 2x\,dy$

and for the equilateral plate $y = \sqrt{3}\,x$ so $x = (1/\sqrt{3})\,y$

and by geometry $s + y = (\sqrt{3}/2)\,a$ so $s = (\sqrt{3}/2)\,a - y$

$$dF = 2\rho g\,[(\sqrt{3}/2)\,a - y][y/\sqrt{3}]\,dy$$

$$\text{So}\quad F = 2\rho g \int_{y=0}^{y=\sqrt{3}/2)\,a} [(\sqrt{3}/2)\,a - y][y/\sqrt{3}]\,dy$$

$F = (1/8)\,\rho\, g\, a^3$ (resulting hydrostatic pressure force on plate)