

Even More Integrals involving Trig Functions

In a Nut Shell: Sometimes you just need to express trig functions in terms of their basic definition. i.e. Tangent is simply sine divided by cosine.

Example $\int \tan x \, dx = \int [\sin x / \cos x] \, dx$

$$u = \cos x \quad du = -\sin x \, dx$$

$$\int \tan x \, dx = -\int du / u \quad \text{which is a standard integral, } (\ln u)$$

$$\int \tan x \, dx = \ln(\cos x) + C$$

In similar manner $\int \cot x \, dx = \int [\cos x / \sin x] \, dx$

$$u = \sin x \quad du = \cos x \, dx$$

$$\int \cot x \, dx = \int du / u \quad \text{which is a standard integral, } (\ln u)$$

$$\int \cot x \, dx = \ln(\sin x) + C$$

In a Nut Shell: Sometimes you may need special tricks such as multiplying and dividing by the same function followed by a substitution.

Example $\int \sec x \, dx$

Multiply and divide $\sec x$ by $(\sec x + \tan x)$

$$\text{and let } u = \sec x + \tan x, \quad du = (\sec x \tan x + \sec^2 x) \, dx$$

$$\text{So integral becomes } \int du / u = \ln |u| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

Use similar strategy for $\int \csc x \, dx$ (don't forget – sign)

$$\int \csc x \, dx = -\ln |\csc x + \cot x| + C$$