Integrals involving Trig Functions

In a Nut Shell: The evaluation of integrals involving trigonometric functions is typically a multi-step process. In many cases you may need to use substitution, trig identities, grouping, and integration by parts.

There are a few general guidelines that help but you need to work many different examples to become proficient. Let's start with one simple example.

Example $I_1 = \int \sin^{9/19} x \cos x \, dx$

Substitution: $u = \sin x$ $du = \cos x \, dx$

So $I_1 = \int u^{9/19} \, du$ which is a standard integral. You need to express the result In terms of the original independent variable, x.

Similar reasoning for the integral: $I_{1a} = \int \cos^{21/19} x \sin x \, dx$

In this case $u = \cos x$ $du = - \sin x \, dx$ and

$I_{1a} = - \int u^{21/19} \, du$ which again is a standard integral.

For integrals of the type: $\int \sin^m x \cos^n x \, dx$
with m an even integer, n an odd integer or vice-versa,

Example $I_2 = \int \sin^8 x \cos^7 x \, dx = \int \sin^8 x \cos^6 x \cos x \, dx$

Now let $\cos^2 x = 1 - \sin^2 x$, $\cos^6 x = (1 - \sin^2 x)^3$

Then $I_2 = \int \sin^8 x (1 - \sin^2 x)^3 \cos x \, dx$

Next let $u = \sin x$, $du = \cos x \, dx$, so $I_2 = \int u^8 (1 - u^2)^3 \, du$

Use a similar strategy for

$I_{2a} = \int \sin^3 x \cos^6 x \, dx = \int \cos^6 x \sin^2 x \sin x \, dx$

Now with $u = \cos x$, $du = -\sin x \, dx$

And using $\sin^2 x = 1 - \cos^2 x$

$I_{2a} = \int \cos^6 x (1 - \cos^2 x) \sin x \, dx = \int \cos^6 x - \cos^8 x) \sin x \, dx$

Let $u = \cos x$ $\int u^6 - u^8) (- du) = \int u^8 - u^6) \, du$ (standard integrals)

Express result in terms of x.