Level Curves, Traces, Intercepts/Visualization of Surfaces

In a Nut Shell: Let $z = f(x,y)$ represent the graph of a function. Then **level curves** are the set of all points (x,y) such that $f(x,y) = c$ where c is a given constant. Each value of c provides a new level curve.

If the function represents temperature, $T = f(x,y)$, then each level curve represents an isothermal. If the function represents altitude, $Z = g(x,y)$, then each level curve represents an isocline. If the function represents pressure, $P = h(x,y)$, then each level curve represents an isobar.

Physical Interpretation of Level Curves Suppose the level curves represent changes in elevation of a surface such as a mountain. Further suppose the level curves change by equal increments (equal changes in elevation). Then in steep areas of the mountain the level curves are closely spaced whereas in fairly level valleys, the level curves may be much farther apart. Closely spaced level curves are an indication of a large gradient of the function.

Traces Another option to assist in sketching a graph is to plot the curves of intersection of the surface with planes parallel to the coordinate planes. **These curves are termed traces**.

A trace may be an intersection of the surface with the coordinate planes or with planes parallel to the coordinate planes. Plot traces on planes. i.e. For the function $z = f(x,y)$. Set $x = 0$ and plot the trace in the yz-plane. Set $y = 0$ and plot the trace in the xz-plane.

Intercepts An intercept is the value of x or y where the surface intersects the x and y-axes. Using intercepts provides a quick check on the location of an edge of the surface on the coordinate axes.

Check for symmetry

Does $f(x,y) = f(-x, -y)$? Symmetric about both axes
Does $f(x,y) = f(-x, y)$? Symmetric about y-axis
Does $f(x,y) = f(x, -y)$? Symmetric about x-axis

Strategy in Visualization of Surfaces

Combine methods using level curves, traces, intercepts, and symmetry to assist in sketching the surface, $f(x,y) = 0$.

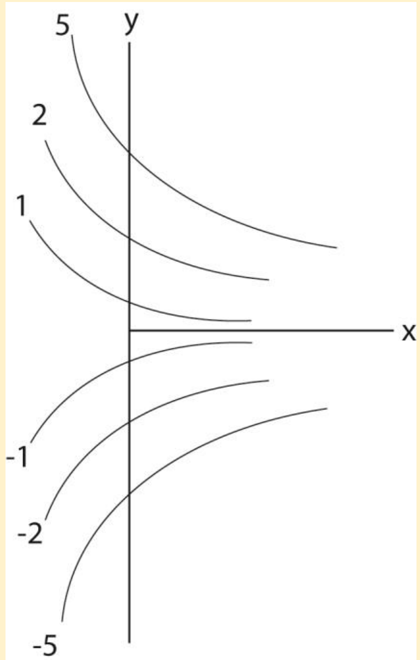
Additionally check the value of $f(x,y)$ at the origin. i.e. $x = y = 0$.

Example: Draw several level curves for the graph of $f(x,y) = y\,e^{2x}$

Strategy: Set $y\,e^{2x} = C$ and let C take on the values 1, 2, 5

So $y(x) = e^{-2x}$, $y(x) = 2\,e^{-2x}$, $y(x) = 5\,e^{-2x}$

Plot each level curve as shown below.



Example: Draw several level curves for the graph of $f(x,y,z) = 36x^2 + y^2 + 36z^2 - 36 = 0$

Strategy:

1. First check for symmetry. Note: $f(x,y,z) = f(-x, -y, -z)$

   Result: The graph of $f(x,y,z)$ is symmetric about all three axes.

Next plot traces.

2. Traces for $x = k =$ constant.

So $y^2 + 36z^2 = 36 - 36k^2$ which are ellipses for $|k| < 1$.

3. Traces for $y = k =$ constant.

So $36x^2 + 36z^2 = 36 - k^2$ which are circles for $|k| < 1$.

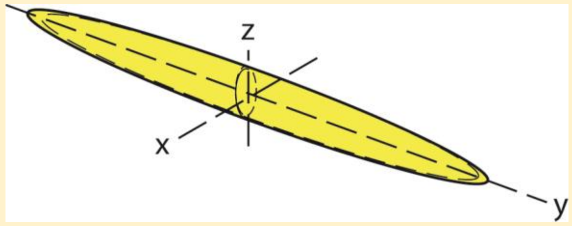
4. Traces for $z = k =$ constant.

So $36x^2 + y^2 = 36 - 36k^2$ which are ellipses for $|k| < 1$.

Result: The graph is an ellipsoid centered at the origin with intercepts at

$x = \pm 1$, $y = \pm 6$, and $z = \pm 1$

See the figure below.



z
x
y