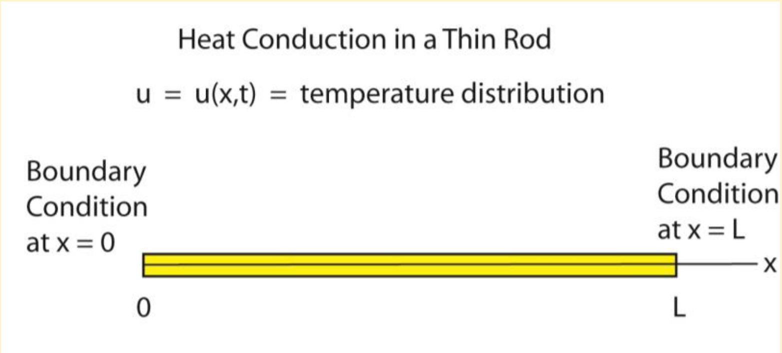
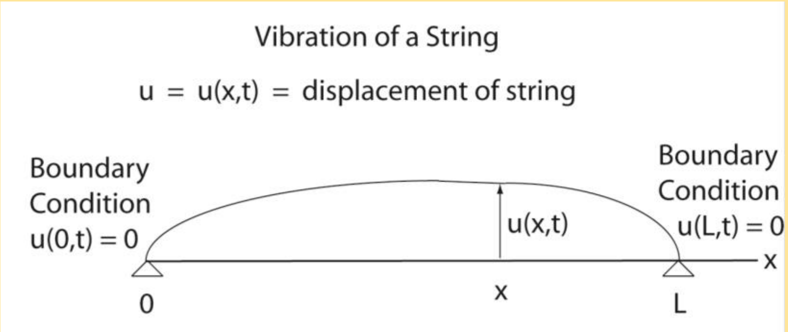
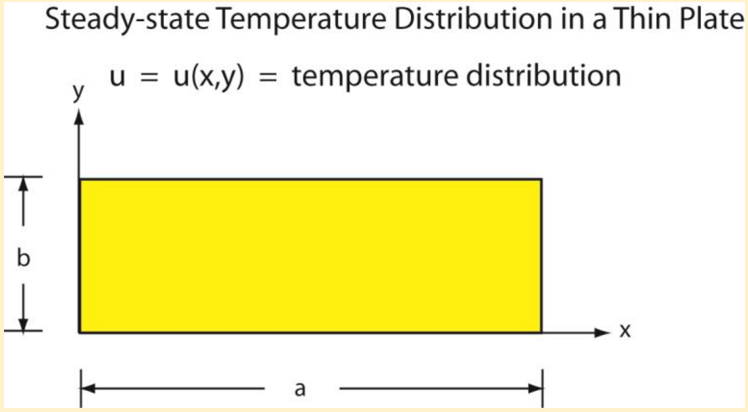
Overview of Boundary Value Applications

In a Nut Shell: There are three common boundary value applications that lend themselves to analysis under the assumption of separation of variables. In addition, suppose each of them have homogeneous boundary conditions. The applications include:

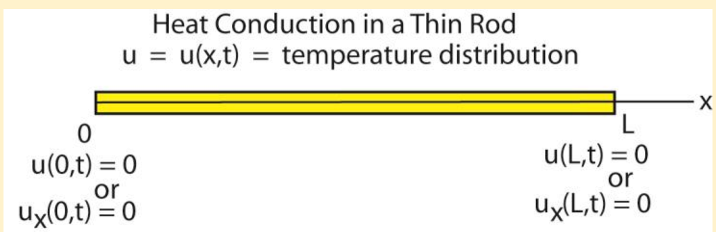
Application	Governing PDE
Heat conduction n a thin rod	$\partial u/\partial t = k\, \partial^2 u/\partial x^2$
Vibrations of a string	$\partial^2 u/\partial t^2 = k\, \partial^2 u/\partial x^2$
Heat conduction in a plate	$\partial^2 u/\partial x^2 + k\, \partial^2 u/\partial y^2 = 0$



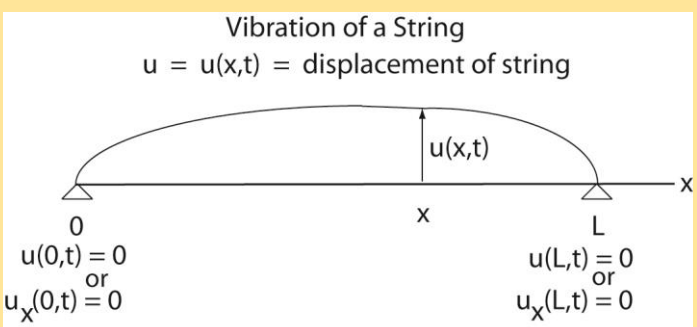




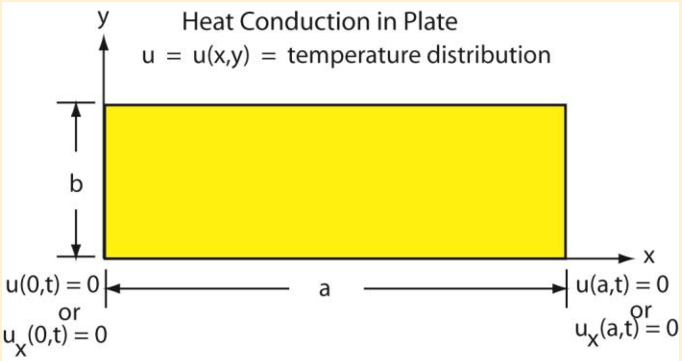
In a Nut Shell: The figures below display the two common types of homogeneous boundary conditions for each application.



$u(0,t) = 0$ and $u(L,t) = 0$ specified temperatures at ends of thin rod
$u_x(0,t) = 0$ and $u_x(L,t) = 0$ ends of rod are insulated



$u(0,t) = 0$ and $u(L,t) = 0$ specified displacements at each end
$u_x(0,t) = 0$ and $u_x(L,t) = 0$ specified gradient of displacement at each end



$u(0,t) = 0$ and $u(a,t) = 0$ specified temperatures at ends of thin plate
$u_x(0,t) = 0$ and $u_x(a,t) = 0$ ends of thin plate are insulated

In a Nut Shell: Homogeneous boundary conditions determine the related eigenvalues and eigenvectors for the applications of heat conduction in a rod, of vibrations of a string, and of heat conduction in a plate. The table below provides a summary. **Note the similarity in each individual application.**

Heat Conduction in a Rod of Length, L

For the homogeneous boundary conditions:
$u(0,t) = 0$ and $u(L,t) = 0$ specified temperatures at ends of thin rod

Expect eigenvalues, $\lambda_n = n^2\pi^2/L^2$ and eigenvectors, $X_n(x) = \sin(n\pi x/L)$

For the homogeneous boundary conditions:
$u_x(0,t) = 0$ and $u_x(L,t) = 0$ ends of rod are insulated

Expect eigenvalues, $\lambda_n = n^2\pi^2/L^2$ and eigenvectors, $X_o(x) = 1$ and $X_n(x) = \cos(n\pi x/L)$

Vibration of a String of Length, L

For the homogeneous boundary conditions:
$u(0,t) = 0$ and $u(L,t) = 0$ specified displacements at ends of string

Expect eigenvalues, $\lambda_n = n^2\pi^2/L^2$ and eigenvectors, $X_n(x)\ (x) = \sin(n\pi x/L)$

For the homogeneous boundary conditions:
$u_x(0,t) = 0$ and $u_x(L,t) = 0$ specified gradients at ends of string

Expect eigenvalues, $\lambda_n = n^2\pi^2/L^2$ and eigenvectors, $X_o(x) = 1$ and $X_n(x) = \cos(n\pi x/L)$

For Heat Conduction in a Plate with dimensions a by b

For the homogeneous boundary conditions:
$u(0,y) = 0$ and $u(a,y) = 0$ temperatures at ends of the plate $x=0$ and $x=a$ are zero

Expect eigenvalues, $\lambda_n = n^2\pi^2/a^2$ and eigenvectors, $X_n(x) = \sin(n\pi x/a)$

For the homogeneous boundary conditions:
$u_x(0,y) = 0$ and $u_x(a,y) = 0$ ends of the plate at $x=0$ and $x=a$ are insulated

Expect eigenvalues, $\lambda_n = n^2\pi^2/a^2$ and eigenvectors, $X_o(x) = 1$ and $X_n(x)$ (x) $= \cos(n\pi x/a)$

Note: If the homogeneous bc's are on the boundaries $y= 0$ and $y = b$, then similar expectations result except exchange b for a with a plate of dimensions a by b .